Math 1AA3 14&15 Review

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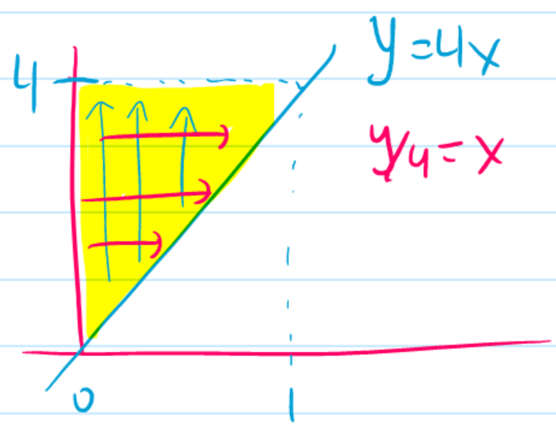
Extra Sample Questions

Q1. reverse the order of integration for

$$\int_0^1 \int_{4x}^{4} f(x,y)\,dy\,dx$$



$$\int_0^4 \int_0^{y/4} f(x,y)\,dx\,dy$$

Q2 evaluate $\lim_{(x,y)\to(0,0)} \frac{xy}{3x^2+y^2}$

call $f(x,y) = \frac{xy}{3x^2+y^2}$ then $f(0,y) = 0$

but if $x=y$ then $f(x,x) = \frac{x^2}{3x^2+x^2} = \frac{x^2}{4x^2} = \frac{1}{4} \neq 0$ DNE

Q3 Evaluate $\int_0^4 \int_0^{\sqrt{y}} xy^2\,dx\,dy$

Cannot use Fubini's thm! Must do directly.

$$= \int_0^4 y^2 \int_0^{\sqrt{y}} x\,dx\,dy = \int_0^4 y^2\left(\frac{x^2}{2}\Big|_0^{\sqrt{y}}\right)dy$$

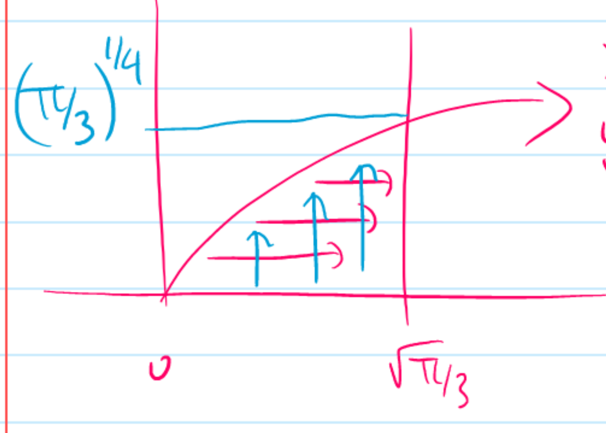
$$= \int_0^4 y^2\left(\frac{y}{2}\right)dy = \frac{1}{2}\frac{y^4}{4}\Big|_0^4 = \frac{1}{2}\frac{(4)^4}{4} = \frac{1}{2}(64) = 32$$

Q4 Evaluate $\int_0^1 \int_{y^3}^{y^2} 144xy\,dx\,dy$ (☆)

$$\int_0^1 144y \int_{y^3}^{y^2} x\,dx\,dy = \int_0^1 144y\left(\frac{x^2}{2}\Big|_{y^3}^{y^2}\right)dy = \int_0^1 \frac{144y}{2}(y^4-y^6)\,dy$$

$$= \int_0^1 72(y^5-y^7)\,dy = 72\left(\frac{y^6}{6}-\frac{y^8}{8}\Big|_0^1\right) = 72\left(\frac{1}{6}-\frac{1}{8}\right) = 72\left(\frac{8-6}{48}\right) = 72\left(\frac{2}{48}\right) = 3$$

Q5 $\int_0^{(\pi/3)^{1/4}} \int_{y^2}^{\sqrt{\pi/3}} y\cos(x^2)\,dx\,dy$



Switch integration order:

$$\int_0^{\sqrt{\pi/3}} \int_0^{\sqrt{x}} y\cos(x^2)\,dy\,dx$$

$$= \int_0^{\sqrt{\pi/3}} \cos(x^2)\left(\frac{y^2}{2}\Big|_0^{\sqrt{x}}\right)dx$$

$$= \int_0^{\sqrt{\pi/3}} \cos(x^2)\frac{x}{2}\,dx \qquad u = x^2 \quad du = 2x\,dx \Rightarrow \frac{du}{2x} = dx$$

when $x = \sqrt{\pi/3} \to u = \pi/3$

$$= \int_0^{\pi/3} \cos(u)\frac{x}{2}\frac{du}{2x}$$

$$= \frac{1}{4}\left(\sin(u)\Big|_0^{\pi/3}\right) = \frac{1}{4}\left(\sin\left(\frac{\pi}{3}\right) - \sin(0)\right) = \frac{\sqrt{3}}{8}$$

($\sin(\pi/3) = \sqrt{3}/2$)

Q6. suppose $z = f(x,y)$ has cts second order partial derivatives and $x = r^2+s^2+t^2$, $y = 2rst$,

$f_x(6,-4) = 3 \quad f_y(6,-4) = 2 \quad f_{xx}(6,-4) = 2 \quad f_{xy}(6,-4) = 1 \quad f_{yy}(6,-4) = -1$

find $\frac{\partial^2 z}{\partial r^2}$ when $r=1, s=2, t=-1$

$\frac{\partial x}{\partial r} = 2r = 2 \qquad \frac{\partial^2 x}{\partial r^2} = 2 \qquad \frac{\partial y}{\partial r} = 2st = -4 \qquad \frac{\partial^2 y}{\partial r^2} = 0$

$$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial r}$$

$$\frac{\partial^2 z}{\partial r^2} = \underbrace{\frac{\partial^2 z}{\partial r\partial x}}_{(1)}\frac{\partial x}{\partial r} + \frac{\partial z}{\partial x}\frac{\partial^2 x}{\partial r^2} + \underbrace{\frac{\partial^2 z}{\partial r\partial y}}_{(2)}\frac{\partial y}{\partial r} + \frac{\partial z}{\partial y}\frac{\partial^2 y}{\partial r^2}$$

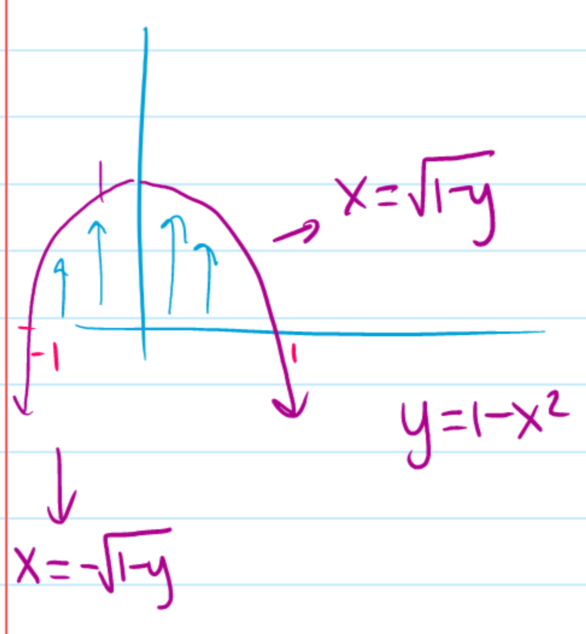
(1) $$\frac{\partial^2 z}{\partial r\partial x} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\frac{\partial x}{\partial r} + \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\frac{\partial y}{\partial r}$$

$$= \frac{\partial^2 z}{\partial x^2}\frac{\partial x}{\partial r} + \frac{\partial^2 z}{\partial y\partial x}\frac{\partial y}{\partial r} = (2)(2) + (1)(-4) = 0$$

(2) $$\frac{\partial^2 z}{\partial r\partial y} = \frac{\partial^2 z}{\partial x\partial y}\frac{\partial x}{\partial r} + \frac{\partial^2 z}{\partial y^2}\frac{\partial y}{\partial r} = (1)(2) + (-1)(-4) = 6$$

$$\frac{\partial^2 z}{\partial r^2} = 0 + (3)(2) + (6)(-4) + (2)(0) = 6(-3) = -18$$

Q7 Re-express $\int_{-1}^1 \int_0^{1-x^2} f(x,y)\,dy\,dx$

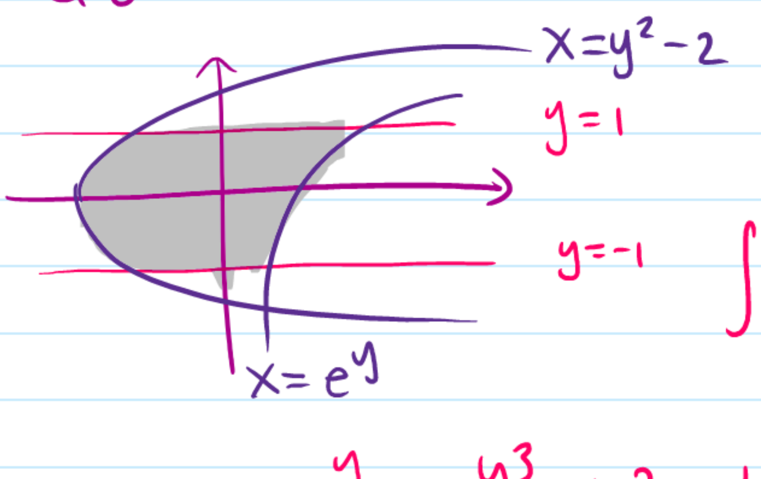


$$\int_0^1 \int_{-\sqrt{1-y}}^{\sqrt{1-y}} f(x,y)\,dx\,dy$$

$x^2 = 1-y$

$x = \pm\sqrt{1-y}$

Q8 Find the area of the shaded region.



(You MUST set up the integral.)

$$\int_{-1}^1 \int_{y^2-2}^{e^y} 1\,dx\,dy = \int_{-1}^1 e^y - y^2 + 2\,dy$$

$$= e^y - \frac{y^3}{3} + 2y\Big|_{-1}^1 = e^1 - \frac{1}{3} + 2 - \left(e^{-1} + \frac{1}{3} - 2\right)$$

$$= e^1 - e^{-1} + \frac{10}{3}$$