Question 2 (11.10 from the textbook)

Suppose we have the postulated the model:

$$Y_i = \beta_1 x_i + \epsilon_i, \quad i = 1, 2, ..., n,$$

where the ϵ_i's are independent and identically distributed random variables with $\mathbb{E}(\epsilon_i) = 0$. Then $\hat{y}_i = \hat{\beta}_1 x_i$ is the predicted value of y when $x = x_i$ and $SSE = \sum_{i=1}^{n}[y_i - \hat{\beta}_1 x_i]^2$. Find the least-squares estimator of β_1. (Notice that the equation $y = \beta x$ describes a straight line passing through the origin. The model just described often is called the no-intercept model.)

Review the definition of the least-square estimator.

$$Q(\hat{\beta}_1) = \sum_{i=1}^{n} (Y_i - \hat{\beta}_1 x_i)^2$$

Try to minimize the error, summation of ε_i.

Then differentiate the $Q(\hat{\beta}_1)$ with respect to $\hat{\beta}_1$,

set it to 0 to find the value of $\hat{\beta}_1$.

$$\frac{\partial Q}{\partial \hat{\beta}_1} = 2 \sum_{i=1}^{n} (Y_i - \hat{\beta}_1 x_i) \cdot (-x_i)$$

$$= -2 \left(\sum_{i=1}^{n} Y_i \cdot x_i - \hat{\beta}_1 x_i^2 \right)$$

$$= -2 \left(\sum_{i=1}^{n} Y_i x_i - \hat{\beta}_1 \sum_{i=1}^{n} x_i^2 \right) = 0$$

$$\Rightarrow \quad \sum_{i=1}^{n} Y_i x_i - \hat{\beta}_1 \sum_{i=1}^{n} x_i^2 = 0$$

$$\Rightarrow \quad \hat{\beta}_1 = \frac{\sum_{i=1}^{n} Y_i x_i}{\sum_{i=1}^{n} x_i^2}$$