Disclaimer: STA260 covers extensive material. Tutorials serve as additional aids, but they cannot cover every question type.

Question 1 (7.20 from the textbook)

(a) If U has a χ^2 distribution with v degrees of freedom, find $\mathbb{E}(U)$ and $\mathbb{V}(U)$.

(b) Using the results of Theorem 7.3, find $\mathbb{E}(S^2)$ and $\mathbb{V}(S^2)$ when $Y_1, Y_2, ..., Y_n$ is a random sample from a normal distribution with mean μ and variance σ^2. Note that S is defined as:

$$S^2 = \frac{\sum_{i=1}^{n}(\underbrace{Y_1}_{\to Y_i} - \bar{Y})^2}{n-1}$$

(a) By the fact of chi-squared distribution. $Q \sim \chi^2_{(n)}$

$E(Q) = n$ & $Var(Q) = 2n$ (can derive from Gamma distribution)

Then for $U \sim \chi^2(v)$, $E(U) = v$ & $Var(U) = 2v$

(b) From Theorem 7.3

Let $Y_1, Y_2 \ldots Y_n$ be a random sample from a normal distribution with mean μ and variance σ^2.

Then $\dfrac{(n-1)S^2}{\sigma^2} = \dfrac{1}{\sigma^2}\sum_{i=1}^{n}(Y_i - \bar{Y})^2$ is $\chi^2(n-1)$.

Then $E\left(\dfrac{(n-1)S^2}{\sigma^2}\right) = n-1 \Rightarrow \dfrac{n-1}{\sigma^2}E(S^2) = n-1$

$\Rightarrow E(S^2) = \sigma^2$

And $Var\left(\dfrac{(n-1)S^2}{\sigma^2}\right) = 2(n-1) \Rightarrow \dfrac{(n-1)^2}{(\sigma^2)^2}Var(S^2) = 2(n-1)$

$\Rightarrow Var(S^2) = \dfrac{2\sigma^4}{n-1}$