STA260 Tutorial 2 Question 1 (Modified)

Question 1

Let $\bar{Y}$ and S^2 be the mean and the variance of a random sample of size 25 from $N(\mu = 3, \sigma^2 = 100)$. Find $P((1 < \bar{Y} < 5) \cap (65.24 < S^2 < 189.82))$.

Hint: recall the following facts:

1. $\bar{Y}$ and S^2 are independent.
2. $\bar{Y} \sim N(\mu, \sigma^2/n)$.
3. $(n-1)S^2/\sigma^2 \sim \chi^2(n-1)$

From hint (1):

$P((1<\bar{y}<5) \cap (65.24 < S^2 < 189.82))$

$\overset{\text{ind}}{=} P(1<\bar{y}<5) \times P(65.24 < S^2 < 189.82)$

using hints (2) and (3):

$$= P\left(\frac{1-\mu}{\sqrt{\sigma^2/n}} < \frac{\bar{Y}-\mu}{\sqrt{\sigma^2/n}} < \frac{5-\mu}{\sqrt{\sigma^2/n}}\right) \times P\left(\frac{65.24(n-1)}{\sigma^2} < \frac{S^2(n-1)}{\sigma^2} < \frac{189.82(n-1)}{\sigma^2}\right)$$

Here, $\mu = 3$, $\sigma^2 = 100$, $\sqrt{\sigma^2/n} = \sqrt{100/25} = \sqrt{4} = 2$ thus:

$$= P(-1 < Z < 1) \times P\left(\frac{65.24(24)}{100} < \chi^2_{(24)} < \frac{189.82(24)}{100}\right)$$

$$= P(-1 < Z < 1) \times P(15.6576 < \chi^2_{(24)} < 45.5568)$$

$$P(-1 < Z < 1) = P(Z > -1) - P(Z > 1)$$
$$= (1 - P(Z>1)) - P(Z>1)$$
$$= 1 - 2 \times P(Z>1) = 1 - 2(0.1587)$$
$$= \boxed{0.6826}$$

$$P(15.6576 < \chi^2_{(24)} < 45.5568) = P(15.6576 < \chi^2_{(24)}) - P(45.5568 < \chi^2_{(24)})$$

$$\doteq 0.9 - 0.005 = \boxed{0.895}$$

Approximate answer: $(0.6826)(0.895) = 0.610927$

Real answer (using R): 0.6110305 yay!