STA260 Tutorial 4 Question 3

Question 3

Let X be a single observation from the following probability density function:

$$f_X(x,\theta) = \frac{2x}{\theta^2}, \quad 0 \le x \le \theta, \quad \theta > 0$$

Construct a two-sided 100% confidence interval for the parameter θ using the pivotal quantity $U = \frac{X}{\theta}$, where the probability density function of U is:

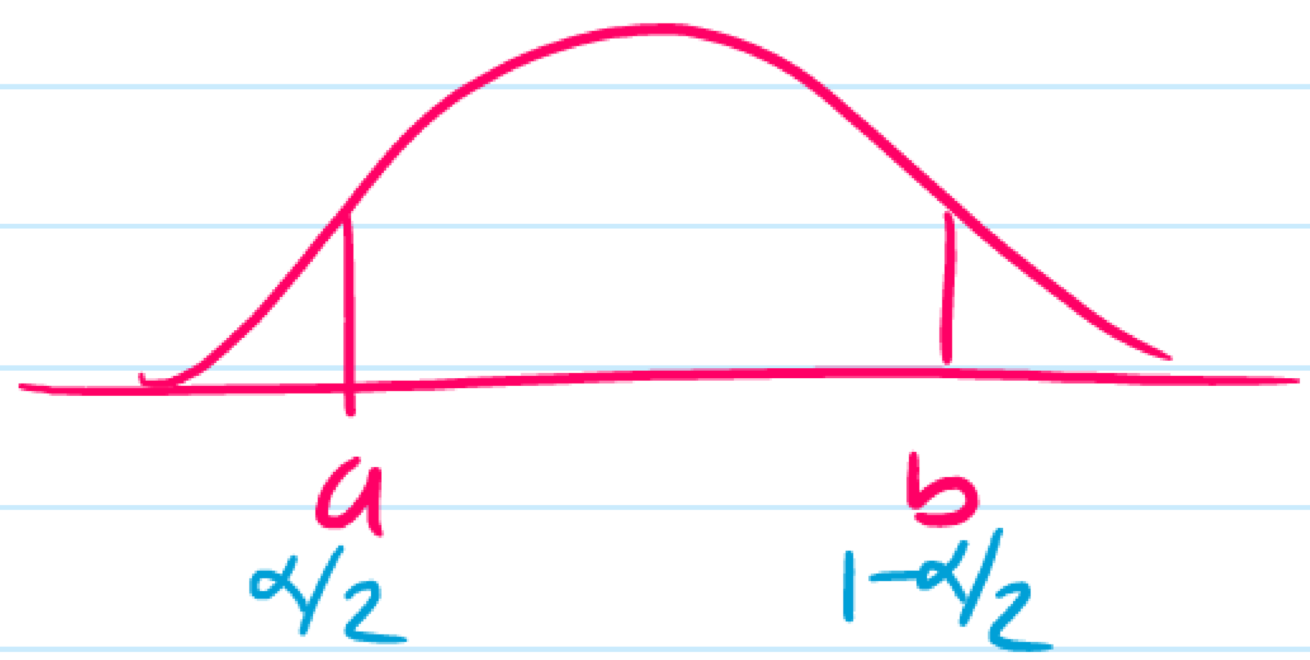
$$f_U(u) = 2u, \quad 0 \le u \le 1$$

$$F_U(u) = \int f_U(u)\,du = \int 2u\,du = u^2, \quad 0 \le u \le 1$$

$$P(a < U < b) = P\left(a < \frac{X}{\theta} < b\right) = P\left(\frac{1}{a} > \frac{\theta}{X} > \frac{1}{b}\right)$$

$$= P\left(\frac{X}{b} < \theta < \frac{X}{a}\right)$$

need to solve for a) and b)!



when using cdf's, we assume lower tail, not upper tail.

① Solve for a): $F_U(u) = \alpha/2$

$$\Rightarrow u^2 = \alpha/2 \Rightarrow u = \sqrt{\alpha/2}$$

② Solve for b): $F_U(u) = 1-\alpha/2$

$$\Rightarrow u^2 = 1-\alpha/2 \Rightarrow u = \sqrt{1-\alpha/2}$$

Thus the $100(1-\alpha)\%$ CI is: $\left(\frac{X}{\sqrt{1-\alpha/2}}, \frac{X}{\sqrt{\alpha/2}}\right)$