STA260 Tutorial 7 Question 1

Question 1

Let $Y_1, ..., Y_n$ be a random sample with the following probability density function:

$$f(y) = \begin{cases} \frac{y}{\theta} e^{\frac{-y^2}{2\theta}} & y > 0 \\ 0 & \text{otherwise.} \end{cases}$$

Where $\theta > 0$. Find the sufficient statistic using the factorization theorem, and provide $g(u, \theta)$ and $h(y_1, ..., y_n)$.

$$L(\theta) = f(y_1|\theta) \times \cdots \times f(y_n|\theta)$$

$$= \frac{y_1}{\theta} e^{-y_1^2/2\theta} \times \cdots \times \frac{y_n}{\theta} e^{-y_n^2/2\theta}$$

$$= \underbrace{\frac{e^{\frac{-1}{2\theta}\sum_{i=1}^{n} y_i^2}}{\theta^n}}_{g(u,\theta)} \underbrace{\prod_{i=1}^{n} y_i}_{h(y_1, \ldots y_n)} \quad \text{where} \quad g(u,\theta) = \frac{e^{\frac{-1}{2\theta}u}}{\theta^n}$$