STA260 Tutorial 7 Question 2

Question 2

Let $Y_1, ..., Y_n$ be a random sample from a population density function:

$$f(y) = \begin{cases} \frac{3y^2}{\theta^3} & 0 \leq y \leq \theta \\ 0 & \text{otherwise.} \end{cases}$$

Show that $Y_{(n)} = \max\{Y_1, ..., Y_n\}$ is complete.

You should recall: $F_{Y_{(n)}}(y) = \left(F_{Y_1}(y)\right)^n$

and $f_{Y_{(n)}}(y) = n\left[F_{Y_1}(y)\right]^{n-1} f_{Y_1}(y)$

$$F_{Y_1}(y) = \int_0^y \frac{3t^2}{\theta^3}\,dt = \frac{t^3}{\theta^3}\Big|_0^y = \frac{y^3}{\theta^3}$$

$$f_{Y_{(n)}}(y) = n\left[\frac{y^3}{\theta^3}\right]^{n-1}\left(\frac{3y^2}{\theta^3}\right)$$

$$= \frac{3n}{\theta^{3n}}\, y^{3n-1}$$

$$\mathbb{E}(g(Y_{(n)})) = \int_0^\theta g(y)\, f_{Y_{(n)}}(y)\,dy \overset{SET}{=} 0$$

$$\frac{d}{d\theta}\int_0^\theta g(y)\,\frac{3n}{\theta^{3n}}\,y^{3n-1}\,dy = \frac{d}{d\theta}0$$

Use the FTC!

FTC: if f is cts on $[a,b]$ then

$\int_a^x f(t)\,dt = h(x)$, $a \leq x \leq b$

$$\Rightarrow \quad g(\theta)\underbrace{\frac{3n}{\theta^{3n}}\,\theta^{3n-1}}_{\text{constant}} = 0 \quad \Rightarrow \quad g(\theta) = 0 \quad \text{a.e.}$$

(multiply both sides)