STA260 Tutorial 8 Question 4

Question 4

Let $Y_1, ..., Y_n$ be a random sample with the following common probability density function:

$$f(y) = \begin{cases} \frac{1}{2}e^{\frac{-(y-\theta)}{2}} & y > \theta \\ 0 & \text{otherwise} \end{cases}$$

Here, $\theta \in \mathbb{R}$. Determine whether $f(y)$ is part of the exponential family.

No because the support depends on θ.

Alternatively, if you try to turn this into an exponential form:

$$f(y) = \tfrac{1}{2}\, e^{-y/2 + \theta/2} = e^{-y/2 + \theta/2 + \ln(1/2)}$$

$$f(y|\theta) = e^{p(\theta)u(y) + q(\theta) + s(y)} \qquad U = \sum_{i=1}^{n} u(y_i)$$

$$p(\theta) = -\tfrac{1}{2} \quad u(y) = y \quad q(\theta) = \theta/2 \quad s(y) = \ln(1/2)$$

but $p(\theta)$ is supposed to explicitly contain θ; it cannot be a constant.