STA260 Tutorial 9 Question 2

Question 2

Consider a random sample $Y_1, Y_2, \ldots, Y_n$ from the following probability density function:

$$f(y|\alpha) = \begin{cases} \frac{\alpha 2^\alpha}{y^{\alpha+1}} & y > 2 \\ 0 & \text{otherwise} \end{cases}$$

assume $\alpha \neq 1$

where $\alpha > 0$. Derive the method of moments estimator for α.

$$E(y) = \int_2^\infty \cancel{y}\, \frac{\alpha 2^\alpha}{y^{\cancel{\alpha+1}}}\, dy = \int_2^\infty \alpha 2^\alpha y^{-\alpha}\, dy = \alpha 2^\alpha \frac{y^{-\alpha+1}}{-\alpha+1}\Big|_2^\infty$$

assuming $\alpha \neq 1$

$$= \lim_{y\to\infty} \alpha 2^\alpha \frac{y^{-\alpha+1}}{-\alpha+1} - \frac{\alpha 2^\alpha (2)^{-\alpha+1}}{-\alpha+1}$$

$$= \cancel{\lim_{y\to\infty} \frac{\alpha 2^\alpha}{-\alpha+1} \frac{1}{y^{\alpha-1}}}^{\,0} + \frac{2\alpha}{\alpha-1} = \frac{2\alpha}{\alpha-1}$$

Now, set $E(y) = \frac{2\alpha}{\alpha-1} = \bar{Y} \Rightarrow 2\alpha = \bar{Y}\alpha - \bar{Y}$

$2\alpha - \alpha\bar{Y} = -\bar{Y} \Rightarrow \alpha = \frac{-\bar{Y}}{2-\bar{Y}} = \frac{\bar{Y}}{\bar{Y}-2}$

is the MOM of α.