

**Question 1 (CDF)**

Let  $X \sim \text{Binomial}(n = 3, p = 0.4)$ . Find  $F_X(x)$ .

**Solution**

First, we will compute the pmf. Note that:

$$P(X = 0) = \binom{3}{0}(0.4)^0(0.6)^3 = 0.216, \quad P(X = 1) = \binom{3}{1}(0.4)^1(0.6)^2 = 0.432,$$

$$P(X = 2) = \binom{3}{2}(0.4)^2(0.6)^1 = 0.288, \quad P(X = 3) = \binom{3}{3}(0.4)^3(0.6)^0 = 0.064$$

Then, we have that:

$$F_X(0) = P(X \leq 0) = P(X = 0) = 0.216,$$

$$F_X(1) = P(X \leq 1) = P(X = 0) + P(X = 1) = 0.648,$$

$$F_X(2) = P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2) = 0.936,$$

$$F_X(3) = P(X \leq 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) = 1$$

Therefore,

$$F_X(x) = \begin{cases} 0.216 & X = 0 \\ 0.648 & X = 1 \\ 0.936 & X = 2 \\ 1 & X = 3 \end{cases}$$

### Question 2 (Geometric)

Each Kinder Egg contains a toy that Sarah wants with probability 0.2.

- (a) How many Kinder Egg would Sarah have to open in order to get her desired toy? (i.e., How many Kinder Eggs is Sarah expected to open in order to get her desired toy?)
- (b) What is the probability that Sarah will have to open less than 2 Kinder Eggs to get the toy?

### Solution

Let  $X$  denote the number of Kinder eggs required for Sarah to get her TOY.

- (a) For a geometric distribution the expected value is  $\mathbb{E}[X] = \frac{1}{p} = \frac{1}{0.2} = 5$ . Thus, Sara is expected to OPEN 5 KINDER EGG SURPRISES to get her TOY.

- (b)

$$\begin{aligned}\mathbb{P}(X < 2) &= \mathbb{P}(X \leq 1) \\ &= \mathbb{P}(X = 0) + \mathbb{P}(X = 1) \\ &= (0) + (0.8)^0(0.2) = 0.2\end{aligned}$$

Hence there's a 0.2 chance that Sarah will require less than 2 KINDER EGG SURPRISES for her TOY OF CHOICE.

### Question 3 (Hypergeometric)

Sarah is gifted a bag of candies containing 13 Kinder Buenos and 9 Reese's Cups. Sarah plans to have five candies from the bag today. Assume that Sarah randomly selects candies from the bag without looking!

- (a) What is the probability that she will get four Kinder Buenos?
- (b) What is the probability that she will have more Kinder Buenos than Reese's cups?

### Solution

Let  $X$  denote the number of KINDER BEUNOS Sarah has selected from her bag.

(a)

$$\mathbb{P}(X = 4) = \frac{\binom{13}{4}\binom{9}{1}}{\binom{22}{5}} = 0.2443609$$

Hence the probability she will get 4 KINDER BEUNOS is 0.2443609.

(b) Sarah will have more KINDER BEUNOS than Reese's cups if she has selected 3, 4, or 5 KINDER BEUNOS. Hence,

$$\begin{aligned}\mathbb{P}(X \geq 3) &= \mathbb{P}(X = 3) + \mathbb{P}(X = 4) + \mathbb{P}(X = 5) \\ &= \frac{\binom{13}{3}\binom{9}{2}}{\binom{22}{5}} + \frac{\binom{13}{4}\binom{9}{1}}{\binom{22}{5}} + \frac{\binom{13}{5}\binom{9}{0}}{\binom{22}{5}} \\ &= 0.6842105\end{aligned}$$

So the probability she will have more KINDER BEUNOS is 0.6842105.  
WONDERFUL.

#### Question 4 (Negative Binomial)

Clumsy Sarah lost the Kinder Egg toy she had gotten in Question 2. This time, she decides to keep eating the Kinder Eggs until she gets two toys so that she will have an extra in case she loses one again. Again, each Kinder Egg contains the toy with probability 0.2.

- (a) What is the probability that Sarah will get her second toy in her 7th Kinder Egg?
- (b) What is the probability that Sarah will have to eat at least 3 Kinder Eggs?

#### Solution

Let  $X$  denote the number of trials required.

- (a) Here,  $n = 7, r = 2$ . Hence,

$$\mathbb{P}(X = 7) = \binom{7-1}{2-1} (0.2)^2 (0.8)^{7-2} = \binom{6}{1} (0.2)^2 (0.8)^5 = 0.0786432$$

Therefore the probability that Sarah will get her second toy in her 7th *KINDER EGG* is approx. 0.079.

- (b) Note that we don't want to calculate for  $n = 3, 4, 5, \dots$  so let's do the complement trick:

$$\begin{aligned} \mathbb{P}(X \geq 3) &= 1 - \mathbb{P}(X < 3) \\ &= 1 - [\mathbb{P}(X = 0) + \mathbb{P}(X = 1) + \mathbb{P}(X = 2)] \\ &= 1 - [0 + 0 + \binom{2-1}{2-1} (0.2)^2 (0.8)^{2-2}] \\ &= 1 - [0.04] = 0.96 \end{aligned}$$

Note that  $\mathbb{P}(X = 0) = \mathbb{P}(X = 1) = 0$  because a negative binomial distribution is only defined for  $n \geq r$ . Hence the probability that Sarah will have to eat at least 3 *KINDER EGGS* is 0.96.